

Low temperature resistivity in a nearly half-metallic ferromagnet

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Abstract

We consider electron transport in a nearly half-metallic ferromagnet, in which the minority spin electrons close to the band edge at the Fermi energy are Anderson-localized due to disorder. For the case of spin-flip scattering of the conduction electrons due to the absorption and emission of magnons, the Boltzmann equation is exactly soluble to the linear order. From this solution we calculate the temperature dependence of the resistivity due to single magnon processes at sufficiently low temperature, namely $k_B T \ll D/L^2$, where L is the Anderson localization length and D is the magnon stiffness. And depending on the details of the minority spin density of states at the Fermi level, we find a $T^{1.5}$ or T^2 scaling behavior for resistivity. Relevance to the doped perovskite manganite systems is discussed.

February 1, 2008

The doped manganite perovskites, $R_{1-x}A_x\text{MnO}_3$, where R is a rare earth element, and A is some divalent ions, exhibit very interesting transport properties. In addition to the well-known phenomenon of “colossal” magnetoresistance at the doping range of around $x \approx 1/3$, it has also been observed that below Curie temperature the electric resistivities can be fitted with a $T^{2.5}$ dependence at the same doping range [1]. A good theoretical description of electron transport for magnetic systems should certainly be able to account for the contributions from the electron-magnon scattering. In conventional metallic ferromagnet, it has long been established [2,3] that the resistivity due to absorption and emission of a single magnon gives rise to a T^2 temperature dependence. The manganite perovskites, however, have been proposed [4] to be almost half-metallic ferromagnets, where the Fermi energy lies near the bottom of the minority spin bands. For a truly half-metallic system, Kubo and Ohata [5] have shown that while single magnon processes are exponentially suppressed by a factor $\exp(-E_g/k_B T)$ where E_g is the minority spin band gap at the Fermi energy, the double magnon processes can lead to a $T^{4.5}$ temperature dependence. The ramification of the fact that this gap may actually be zero and the Fermi energy passes through a region of low density of states in the minority spin channel [4] in these manganites, however, is still left unexplored.

In this Letter, we propose a model for the electron transport in this type of nearly-half metallic ferromagnet (NHMF). In this model, there is no true band gap at the Fermi energy for the minority spin electrons. Instead, the minority spin electrons do not conduct current due to Anderson-localization driven by disorder. Therefore, the system is half-metallic as far as transport is concerned. But there is a finite electronic density of states at the Fermi energy for the minority spin. This allows spin-flip scatterings involving only single magnons to occur.

The relevant magnon contributions at low temperatures are from the long wavelength magnons, for which the energy dispersion can be accurately de-

scribed by $\omega(\mathbf{q}) = Dq^2$, with D being the stiffness. The squared matrix element for the absorption or emission of a long wavelength magnon with wavevector \mathbf{q} is given by,

$$M_l(\mathbf{k}, \mathbf{q}) = \frac{2J^2S}{N^2} |\phi_l(\mathbf{k} - \mathbf{q})|^2 \approx \frac{2J^2S}{N^2} \left(\frac{L}{a}\right)^3 \exp(-L^2|\mathbf{k} - \mathbf{q}|^2), \quad (1)$$

where N is total number of lattice sites, $\phi_l(\mathbf{k})$ are the Fourier components of the localized minority spin electron wavefunctions, ϕ_l , the factor $2J^2S$ comes from ferromagnetic s-d type interaction, a^3 is the volume of crystal unit cell, and L is the Anderson localization length. In the limit of $L \rightarrow \infty$, there is no Anderson localization, and $M(\mathbf{k}, \mathbf{q})$ simply gives a delta function corresponding to the momentum conservation. On the other hand, if L is sufficiently small, or alternatively the temperature is sufficiently low, so that $k_B T \ll D/L^2$, then the \mathbf{k} -dependence of $|\phi_l(\mathbf{k})|^2$ is weak, and the matrix elements $M_l(\mathbf{k}, \mathbf{q})$ can be approximated by a constant

$$M_l = \frac{\mu_l}{N^2} \approx \frac{2J^2S}{N^2} \left(\frac{L}{a}\right)^3 \exp(-L^2 k_F^2) \quad (2)$$

for each localized state.

We wish to emphasize the importance of Anderson localization to our NHMF model. The itinerant model obtained from our model by taking the $L \rightarrow \infty$ limit is *very* different from the model used in Refs. [2,3] where no conduction electron exchange splitting is considered. In our case, without Anderson localization, an additional delta function for the momentum conservation will force the magnon energy to be at least E_{stoner} , the Stoner threshold. This suppresses the single magnon process by a factor of $\exp(-E_{\text{stoner}}/k_B T)$. Without Anderson localization, the phase space that satisfies both the energy and the momentum conservation is exponentially small ($E_{\text{stoner}} > 0$) even when the minority spin band gap at Fermi energy is zero. Therefore, Anderson localization of minority spin electrons is necessary for the contribution of the single magnon process to be significant to the low temperature resistivity. We show later that the Anderson localization also allows the exact solution of the model to linear order.

Let us now turn to the Boltzmann equation for majority spin electrons,

$$-e\mathbf{v}_1(\mathbf{k})F_1'[E_1(\mathbf{k})] \cdot \mathcal{E} = \left(\frac{dF_1[E_1(\mathbf{k})]}{dt} \right)_{\text{col}}, \quad (3)$$

where $e = |e|$, $\hbar\mathbf{v}(\mathbf{k}) = \nabla_{\mathbf{k}}E_1(\mathbf{k})$, \mathcal{E} is the applied external field, and $F_1[E_1(\mathbf{k})]$ is the distribution function of the majority spin electrons, and the subscript 1 denotes the majority spin. To calculate the collision term for the single magnon spin-flip processes, we use the low temperature limit of $M_l(\mathbf{k}, \mathbf{q})$, Eq. (2). The collision term is thus

$$\begin{aligned} \left(\frac{dF_1[E_1(\mathbf{k})]}{dt} \right)_{\text{col}} = & \frac{2\pi}{\hbar} \sum_l \sum_{\mathbf{q}} \{ F_2[E_1(\mathbf{k}) + \omega(\mathbf{q})](1 - F_1[E_1(\mathbf{k})])(n[\omega(\mathbf{q})] + 1) \\ & - F_1[E_1(\mathbf{k})](1 - F_2[E_1(\mathbf{k}) + \omega(\mathbf{q})])n[\omega(\mathbf{q})] \} M_l \delta[E_l - E_1(\mathbf{k}) - \omega(\mathbf{q})], \end{aligned} \quad (4)$$

where the first term is due to the emission of a magnon by a minority spin electron and the second term is due to the absorption of a magnon by a majority spin electron, and the subscript 2 denotes the minority spin. Here $n(x) = 1/[\exp(x/k_B T) - 1]$ is the Bose-Einstein distribution function, $F_2(x)$ is the distribution function for the minority spin electrons, and E_l is the energy of the localized minority state ϕ_l .

Because ϕ_l 's are localized, the distribution function for the minority spin electrons are unchanged by the external field to the linear order in \mathcal{E} [6], i.e., $F_2(x) = f(x)$, where $f(x) = 1/\{\exp[(x - E_F)/k_B T] + 1\}$ is the equilibrium Fermi distribution function, and E_F is the Fermi energy. As is shown below, the fact that the minority spin distribution is unchanged to linear order allows an exact linear order solution of the Boltzmann equation

We define

$$-e\Delta(\mathbf{k}) = \{F_1[E_1(\mathbf{k})] - f[E_1(\mathbf{k})]\}, \quad (5)$$

so that the current is given by

$$\mathbf{j} = \frac{e^2}{8\pi^3} \int d\mathbf{k} \mathbf{v}_1(\mathbf{k}) \Delta(\mathbf{k}). \quad (6)$$

Thus Eq. (4) gives

$$\left(\frac{dF_1[E_1(\mathbf{k})]}{dt}\right)_{\text{col}} = \frac{2\pi}{\hbar} \sum_l \sum_{\mathbf{q}} e\Delta(\mathbf{k}) M_l \frac{f[E_1(\mathbf{k}) + \omega(\mathbf{q})]}{f[E_1(\mathbf{k})]} \exp\left[\frac{\omega(\mathbf{q})}{k_B T}\right] n[\omega(\mathbf{q})] \delta[E_l - E_1(\mathbf{k}) - \omega(\mathbf{q})], \quad (7)$$

Combining Eqs. (3) and (7), and keeping the terms linear to the external field, we get

$$-\mathbf{v}_1(\mathbf{k}) f'[E_1(\mathbf{k})] \cdot \mathcal{E} = \frac{2\pi}{\hbar} \sum_l \sum_{\mathbf{q}} M_l \Delta(\mathbf{k}) \frac{f[E_1(\mathbf{k}) + \omega(\mathbf{q})]}{f[E_1(\mathbf{k})]} \exp\left[\frac{\omega(\mathbf{q})}{k_B T}\right] n[\omega(\mathbf{q})] \delta[E_l - E_1(\mathbf{k}) - \omega(\mathbf{q})]. \quad (8)$$

Upon solving for $\Delta(\mathbf{k})$, we find,

$$\Delta(\mathbf{k}) = -\frac{\hbar}{2\pi} \mathbf{v}_1(\mathbf{k}) f'[E_1(\mathbf{k})] \cdot \mathcal{E} \left\{ \frac{\bar{\mu}}{N} \sum_{\mathbf{q}} \frac{f[E_1(\mathbf{k}) + \omega(\mathbf{q})]}{f[E_1(\mathbf{k})]} \exp\left[\frac{\omega(\mathbf{q})}{k_B T}\right] n[\omega(\mathbf{q})] g_2[E_1(\mathbf{k}) + \omega(\mathbf{q})] \right\}^{-1}, \quad (9)$$

where we have assumed that M_l are not very different for different localized states, with $\bar{\mu}$ being the average of the matrix elements μ_l , and $g_2(E) = \frac{1}{N} \sum_l \delta(E_l - E)$ for the minority density of states. We define variables $x = [E_1(\mathbf{k}) - E_F]/k_B T$ and $y = \omega(\mathbf{q})/k_B T$, to simplify the above equation into

$$\Delta(\mathbf{k}) = \frac{\hbar}{2\pi} \mathbf{v}_1(\mathbf{k}) \frac{1}{k_B T} \frac{\exp(x)}{[\exp(x) + 1]^3} \cdot \mathcal{E} \left\{ \frac{\bar{\mu}}{4\pi^2} \left(\frac{k_B T}{D/a^2}\right)^{1.5} I(x, k_B T) \right\}^{-1}. \quad (10)$$

where

$$I(x, k_B T) = \int_0^\infty dy \sqrt{y} \frac{g_2[E_F + k_B T(x + y)]}{[\exp(x + y) + 1][1 - \exp(-y)]}. \quad (11)$$

Multiplying the above equation by $\mathbf{v}(\mathbf{k})$ and integrating over \mathbf{k} , and also noting that the integrand is sharply peaked at $E_1(\mathbf{k}) = E_F$, we get (cf Eq.6)

$$\rho = \frac{3\bar{\mu}a^3}{2\pi\hbar e^2 g_1(E_F) v_1^2(E_F)} \left(\frac{k_B T}{D/a^2}\right)^{1.5} \left\{ \int_{-\infty}^\infty dx \frac{\exp(x)}{[\exp(x) + 1]^3 I(x, k_B T)} \right\}^{-1}, \quad (12)$$

where the majority spin density of states $g_1(E)$ is defined as,

$$g_1(E) = \frac{a^3}{8\pi^3} \int d\mathbf{k} \delta[E - E_1(\mathbf{k})]. \quad (13)$$

We assume that the minority density of states, g_2 , scales near the Fermi energy as, $g_2(E_F + E) \sim E^\alpha$. This leads to the scaling behavior of the resistivity, $\rho \sim T^{1.5+\alpha}$.

It seems that the sensible possibilities for α in our NHMF model is either zero or 0.5. The former corresponds to a finite density of states for minority spin electrons (even though they are localized) and the latter to the situation that the Fermi surface just touches the edge of the minority spin bands, and it is assumed that Anderson localization does not change the density of states profile. Thus we find that in the low temperature limit, i.e., $k_B T \ll D/L^2$, the resistivity due to single magnon scattering scales with the temperature with an exponent between 1.5 and 2. The $T^{2.5}$ temperature dependence reported experimentally [1] seems to correspond to a higher temperature range, and thus may not be dominated by the single magnon processes. However, since the temperature dependence of the contribution from the single magnon processes has a lower exponent than electron-electron scattering (T^2) or two magnon processes ($T^{4.5}$), single magnon scattering may become important at a sufficiently low temperature.

The disorder in the NHMF materials such as doped perovskite manganites is due to either Mn-O-Mn bond bending or static spin canting [7]. When an external magnetic field is applied, the disorder is expected to be reduced. Consequently, the Anderson localization length will increase with the applied magnetic field. This in turn reduces the matrix element M for the single magnon scattering (cf Eq.1), and reduces the resistivity of the material. Thus even at low temperature, these NHMF materials should also have a large magnetoresistance.

XDW thank Dr. Jiandi Zhang for bringing my attention to Ref [1]. This research is sponsored by the U.S. Department of Energy under Contract DE-AC05-96OR22464 with Lockheed Martin Energy Research Corporation, and by the University of Virginia under interagency agreement DOE No. ERD-96-XJ003.

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